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Geocoding as an Inverse Problem

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Defence Research Establishment Atlantic

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Geocoding As An Inverse Problem

John A. Fawcett

Abstract:

In this paper we consider the construction of the seabed reflectivity given overlapping sidescan sonar beam coverage of the seabed.

Résumé :

Le présent article porte sur la reconstruction de la réflectivité du fond marin obtenue en utilisant des faisceaux redondants des sonars à balayage latéral.

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GEOCODING AS AN INVERSE PROBLEM

by

John A. Fawcett

INTRODUCTION: For route survey applications, modern multibeam sidescan sonars are used. A multibeam sidescan sonar is designed to provide contiguous coverage of the seabed up to some maximum towfish speed. Often, however the sonar is towed at speeds significantly lower than this maximum and the multiple beams of the sonar from sucessive pings overlap each other (redundant coverage). Beams which are judged to be redundant are often discarded in sonar processing and display. However, they in fact, often sample slightly different segments of the seabed and hence by appropriate processing can be used to effectively improve the along-track resolution of the sonar.

PRINCIPAL RESULTS: It is shown that linear or non-linear processing of redundant sidescan sonar beam coverage can be used to yield an along-track resolution better than the beamwidth of the system. However, for noisy data some constraints need to be built into the solution to yield acceptable results.

SIGNIFICANCE OF RESULTS: The results of this paper indicate that by appropriate processing of sidescan sonar multibeam data it is possible to improve the resolution of the system. This could lead to improved detection and classification of minelike objects.

FUTURE PLANS: There are number of areas of future research for this method: (1) the optimal choice of a cost function to yield higher-resolution, yet sufficiently smooth, seabed reflectivity values (2) faster methods of optimization (3) the performance of the method with a variety of real data examples.

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GÉOCODAGE COMME UN PROBLÈME INVERSE

Par

John A. Fawcett

INTRODUCTION : Pour les applications de levés des fonds marins, on utilise des sonars multi-faisceaux à balayage latéral qui sont conçus de manière à offrir une couverture contiguë du fond marin jusqu'à la vitesse maximale du corps remorqué. Toutefois, il arrive souvent que le sonar soit remorqué à des vitesses vraiment inférieures à la vitesse maximale et que les faisceaux multiples du sonar se chevauchent à la suite d'impulsions successives (couverture redondante). Les faisceaux jugés redondants sont souvent rejetés lors du traitement et de la présentation des signaux de sonars. Cependant, ils échantillonnent souvent des segments légèrement différents du fond marin et, de là, il est possible, à l'aide du traitement approprié, d'améliorer efficacement la définition longitudinale du sonar.

RÉSULTATS PRINCIPAUX : Pour que la définition longitudinale soit supérieure à la largeur du faisceau du système, il est prouvé que le traitement linéaire ou non linéaire de la couverture redondante des faisceaux par des sonars à balayage latéral est préférable. Toutefois, pour les données bruitées, certaines contraintes doivent être ajoutées afin de donner des résultats acceptables.

IMPORTANCE DES RÉSULTATS : Les résultats indiquent qu'à l'aide du traitement approprié des données recueillies par les sonars multi-faisceaux à balayage latéral, il est possible d'améliorer la définition du système. Ce traitement approprié permettrait d'améliorer la détection et la classification des objets pouvant être confondus avec des mines.

PROJETS FUTURS : Il y a un certain nombre de domaines permettant d'approfondir la recherche pour cette méthode : (1) le choix optimal d'une fonction de coût pour obtenir une meilleure définition, à savoir une définition plus uniforme des valeurs de réflectivité du fond marin, (2) des méthodes plus rapides d'optimisation, (3) la performance de la méthode ainsi qu'une grande diversité d'exemples de données réelles.

INTRODUCTION

For a sidescan sonar, the measured backscattered time series for the various beams is related to an integral of the seabed reflectivity in both the cross-track and along-track directions (i.e. the footprint the beam makes on the bottom during a specified time interval). This integral would, in general, be weighted by the beampattern. For many multibeam sidescan sonars, the beampattern in the nearfield is approximated by a straight beam with a specified width such as, for example, 20 cm. Most multibeam sidescan sonars are configured so that at maximum tow speed, the sonar will advance almost one physical length during a ping cycle; in this case all beams are used to provide contiguous coverage of the seabed. However, often in surveys, the tow speed will be significantly less than the maximum and there will be redundant coverage by the beams. That is, a portion of the seabed may be covered by a beam from more than one beamset. The concept of improving image quality by utilizing the "redundant" beams, instead of ignoring them as is often done, was presented in the work of Huff [1].

In this paper we first show that the part of the geocoding process which is related to interpreting the actual beam measurements as seabed reflectivity on a georeferenced grid can be posed as an inverse problem and second, we show that the redundant beam coverage can, in fact, improve the along-track resolution of the sonar system.

I. THEORY

We start by making a simplifying assumption: we will consider the beam time series for a fixed cross-track range and consider only along-track variations in the reflectivity at this particular cross-track range (time) (i.e., we are considering the seabed reflectivity as a one-dimensional profile). In this paper, reflectivity is taken to mean the amount of backscatter emanating from that point on the seabed and includes all the effects of impedance, roughness, and the angle of the incident energy. Let us denote a beam measurement, with the beam centred at x as $B(x)$. This function is the convolution of the beampattern $U(x)$ with the reflectivity profile $R(x)$ or

$$B(x) = U * R \tag{1}$$

In fact, we only measure $B(x)$ at a finite number of positions x_i and so it is only possible to approximately recover R from the measurements $B(x_i), i = 1, ..., N$. As can be seen from Eq.(1) one could approach the inverse problem as a deconvolution problem in the spectral domain. Instead,

here, we use a linear algebra approach. Let us consider $0 \leq x \leq L$ and consider the reflectivity profile to be expressed in the form

$$R(x) = \sum_{j=1}^{M} a_j S_j(x) \tag{2}$$

where $S_j(x)$ are step functions and may have constant or non-constant lengths. The coefficients a_j are to be determined. With such a representation it is straightforward to write that

$$\vec{b} = \Omega \vec{a} \tag{3}$$

where $\vec{b}$ represents the beam measurements, $\vec{a}$ are the unknown coefficients and Ω is an $N \times M$ matrix relating the two. The element Ω_{ij} is the integral projection of the ith beampattern onto the jth reflectivity step, i.e.,

$$\Omega_{ij} = \int_0^L U(x; x_i) S(x; x_j) dx \tag{4}$$

where $U(x; x_i)$ denotes the beampattern centred at $x = x_i$ and $S(x; x_j)$ is the reflectivity step centred at $x = x_j$. Since we take $U(x; x_i)$ and $S(x; x_j)$ to be of finite extent, then Ω_{ij} will have a banded structure.

In the case that Ω is a square matrix it is possible to exactly solve for the coefficient vector $\vec{a}$; in general, for $M \neq N$ we will have either an overdetermined or underdetermined system of equations. In addition it is possible to add additional equations in an attempt to constrain the solutions to appear more physically reasonable. As we shall see there are cases where the system of equations of (3) are ill-conditioned. To a certain extent, the conditioning of the system depends upon how we choose the $S_j(x)$ to represent the reflectivity profile. We now consider some numerical examples.

The traditional approach to assigning a reflectivity value to an interval would be to perform a weighted average of the beamdata that corresponded to that interval. In particular we could weight the beamdata, b_i/δ (δ is the beamwidth) by the length of the interval that the beampattern overlaps the profile interval S_k. This is overlap is the matrix element Ω_{ik} or

$$\hat{a}_k = \sum_{i=1}^{N} \Omega_{ik} b_i / \delta \tag{5}$$

To take out the effect of the total length of the intervals related to a measurement, we compute

$$\tilde{a}_k = \frac{\hat{a}_k}{\sum_{i=1}^{N} \Omega_{ik}} \tag{6}$$

Here we have used the superscript $\tilde{a}_k$ to denote that this is an approximation. In fact, $\sum_{i=1}^{N} \Omega_{ik}$ is equal to δ except for the first and last values of k. The equation for $\tilde{a}_k$ can be written in the form

$$\tilde{a}_k = \frac{\Omega^T \vec{b}}{\delta \times \Omega^T \vec{1}} \tag{7}$$

Below we will refer to this approximation as the backpropagation approximation.

II. NUMERICAL EXAMPLES

A. One-dimensional profile

We will assume a sidescan sonar which is physically 1 m in length and produces 5 beams of 20 cm width. The sonar is towed at a speed of 6.4585 knots and has a ping rate of 0.15 seconds. This corresponds to the sonar moving ahead 0.5 m during a ping cycle. We will take the starting position of the sonar to be such that the first beam is centred at x =0.1 m (i.e., this beam just covers up to $x = 0$). In Fig. 1 we show the beampositions for the first 25 beams of 250 beams (50 ping cycles).

There is 0.1 m offset coverage from the 0.5 m position to 25 m for a total of 246 beams providing this spatial sampling. Let us parameterize the reflectivity profile with step functions spanning the intervals $(.4,.5),(.5,.6),...(24.9,25.0)$; this results in 246 steps. The beam measurement centred at 25.0 m also involves the profile from $(25.0,25.1)$ and hence we need to make an assumption about this in order to close the system of equations. A reasonable assumption is to set this interval value equal to that of $(24.9,25.0)$. With this partitioning, the matrix Ω has the form

$$\begin{pmatrix} 0.1 & 0.1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0.1 & 0.1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0.1 & 0.1 & 0 & 0 & \cdots \\ \vdots & & & & & \vdots & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 \end{pmatrix} \tag{8}$$

Thus this is a bi-diagonal matrix and can be inverted very quickly, in fact by explicit recursion starting with the last element. We consider a reflectivity profile which has a highlight 0.1 m in width, which is half the sonar beamwidth, followed by a shadow; the background is given by a sinusoidal variation, so that the end condition that the reflectivity is a constant is not exactly met. The beamdata, after the beams have been sorted according to position is shown in Fig. 2 (and we use only those corresponding to a 0.1 m spacing). The resulting reconstructed profile (solid and blocky)and the original profile (dashed) are shown together in Fig. 3. As can be seen, the reconstruction is excellent (note that there is an approximate scaling from the beam data of Fig. 2 to that of Fig. 3 due to the beamwidth of 0.2 m) .

We now add Gaussian noise with standard deviation 0.01 which is approximately 10% of the average amplitude to the original beam data resulting in the beamdata of Fig. 4. The resulting inversion result is shown in Fig. 5 - it is a very unacceptable result (here, the actual values of the solution are shown - they have not been converted into the profile) . However, if we restrict the

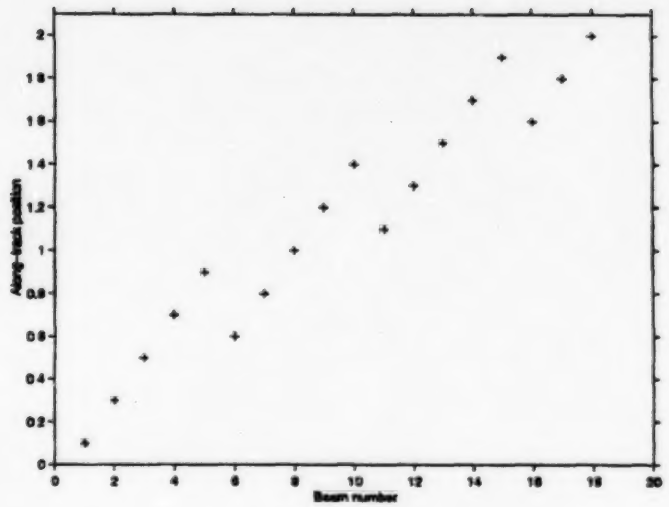

Figure 1: Beam coverage positions

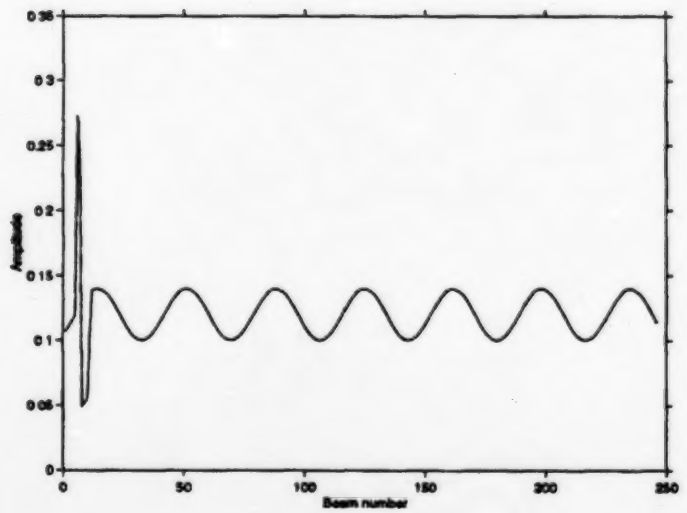

Figure 2: Beam data for specified reflectivity profile

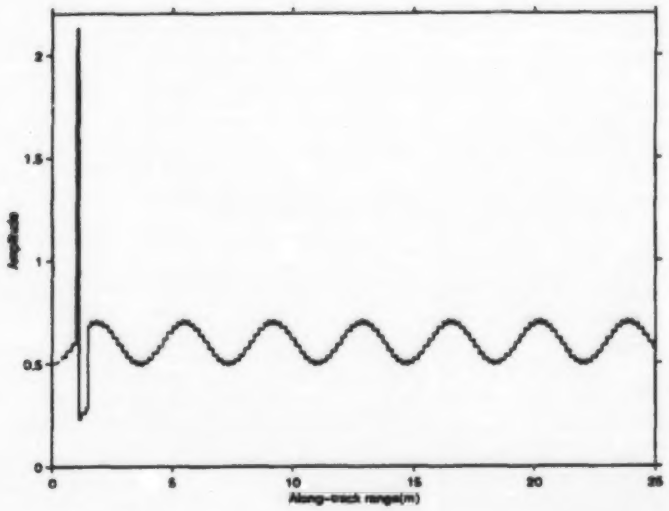

Figure 3: The original profile(dashed) and reconstructed profile(solid)

singular values of the matrix to be greater than 0.05 then the resultant inversion is shown in Fig.6. It shows good resolution for the highlight but is somewhat inaccurate in amplitude at the peak and is very oscillatory elsewhere. In Fig. 7 we show the results of using the backpropagation method; the highlight region has been smoothed over, but the overall reconstruction is much more "pleasing". In the next two figures we show the results of the inversion for a different profile, but with the same noise levels added to the beam data. Here the inverted results are very good for the highlight region and somewhat oscillatory for the background. The backpropagation solution is, once again, smoother including the highlight region where the details of the structure have been smoothed over.

It may be possible to improve the inversion results by either filtering the original data or filtering the inverted results. For example, one strategy which sometimes is successful is to compare the inverted solution with the backpropagated solution and merge the two solutions by keeping the inverted solution when it differs from the backpropagated solution by some threshold. Everywhere else the profile is specified by the smooth backpropagated solution. The approach we take, however, is to impose a non-linear constraint and use an iterative technique to find a solution. In particular we wish to minimize the functional

$$\vec{f}(\vec{x}) = |\Omega\vec{x} - \vec{b}| + (\sum_{j=1}^{N-1} |x_{j+1} - x_j|)^{1/2}/\sigma. \tag{9}$$

The use of the square root in the second terms of Eq.(9) is in order to encourage large jumps in reflectivity in comparison to several small ones. In our examples, we found that $\sigma = 1000$ or 2000 for

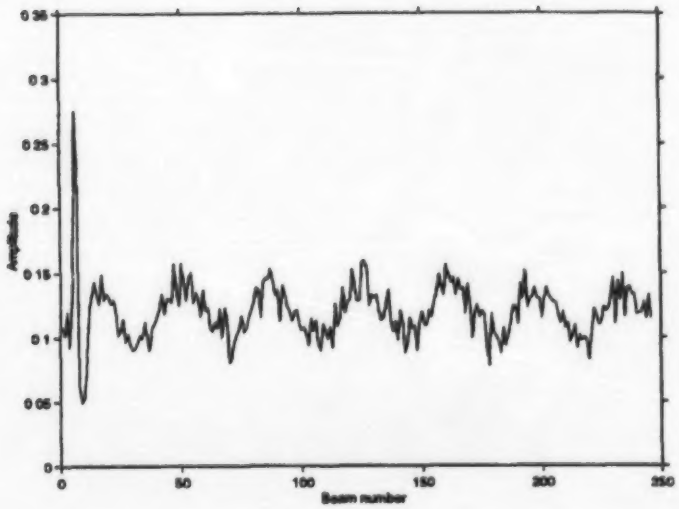

Figure 4: Noisy beam data

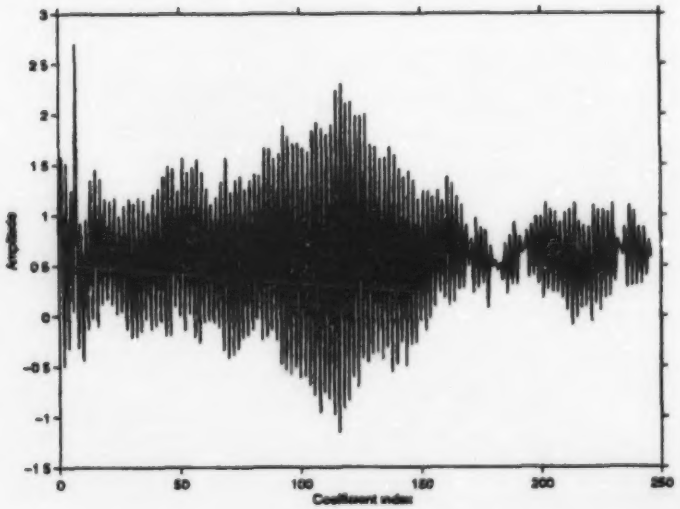

Figure 5: Inversion of noisy beam data

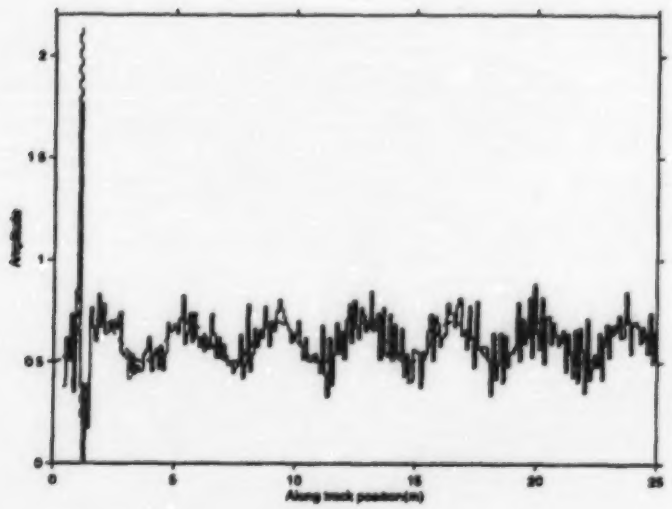

Figure 6: Inversion of noisy beam data with singular values truncated at 0.05

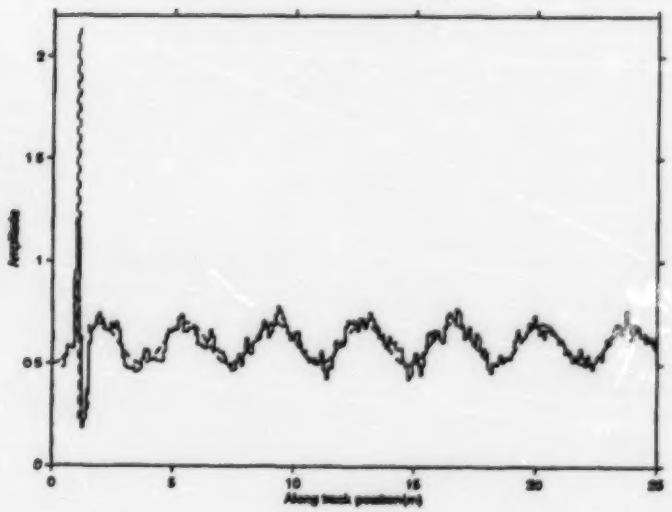

Figure 7: Inversion of noisy beam data using "backpropagation"

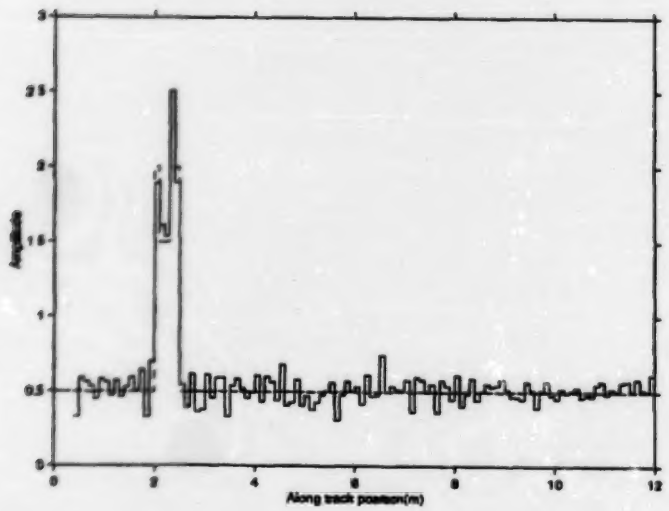

Figure 8: Inversion of noisy beam data with singular values truncated at 0.05 (dashed is input profile, solid is reconstruction)

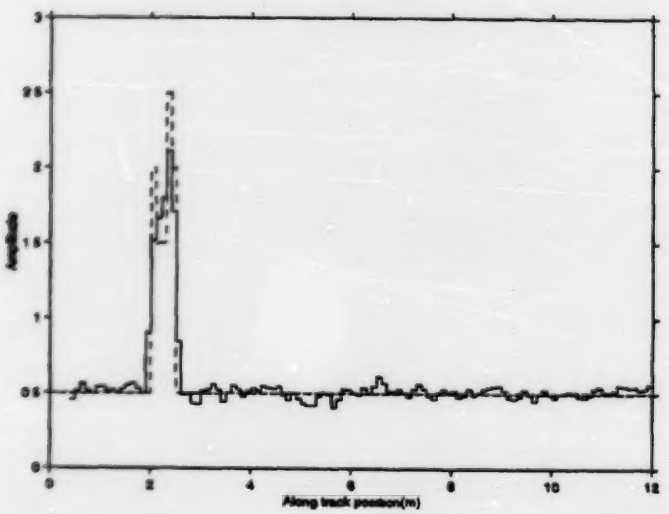

Figure 9: Inversion of noisy beam data using "backpropagation" (dashed is input profile, solid is reconstruction)

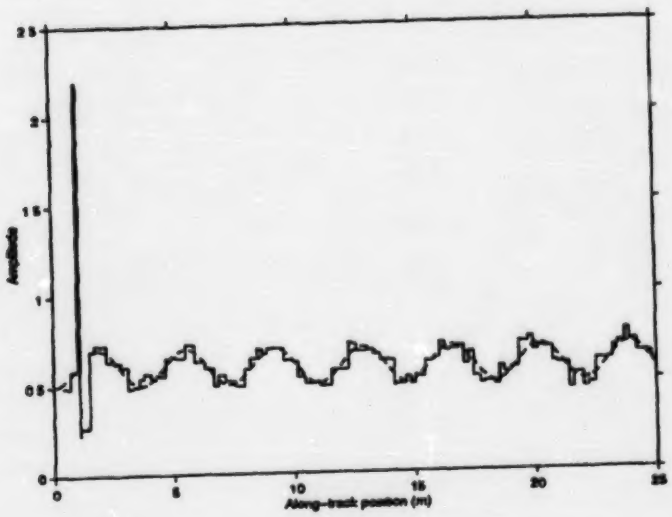

Figure 10: Inversion of noisy beam data using an iterative, non-linear reconstruction

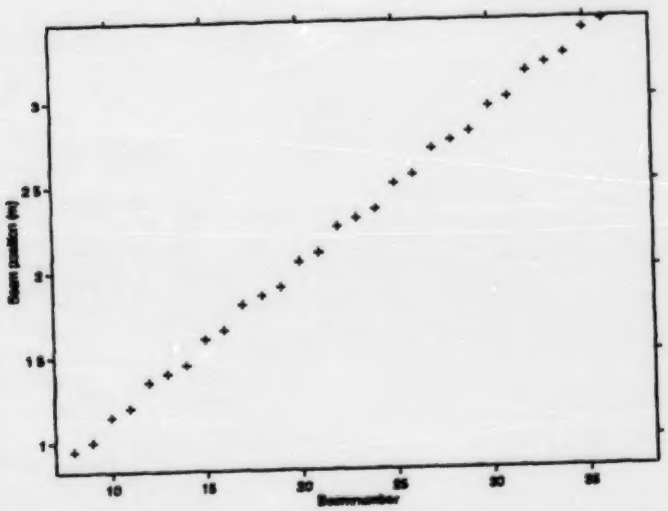

Figure 11: The beampositions (centre of beam) as a function of beamnumber for a shift of 0.45m between pings

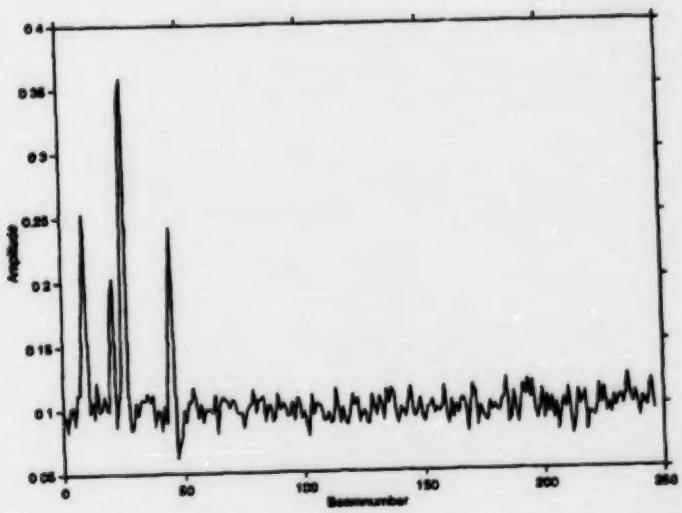

Figure 12: The noisy beam data for the case of 0.45m between pings

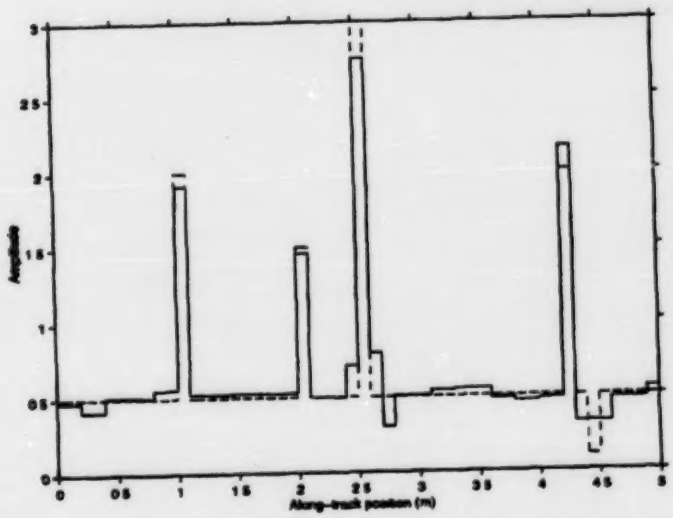

Figure 13: The reconstructed (solid) and input profiles (dashed)

the second term was a good choice. We choose a very simplistic iteration, we just consider random perturbations of the parameters and keep those which decrease the value of $J(\vec{x})$. We repeat the reconstruction using the noisy beam data for the first profile. The result is shown in Fig.10. The reconstructed results are excellent - we have obtained the fine resolution, yet suppressed the high frequency oscillations.

So far, we have taken the towbody to move exactly 0.5m between pings - resulting in an uniform (except at the beginning and end of the data set) beam spacing of 0.1m. We now consider a case where the towbody moves 0.45 m between pings; this results in a non-uniform beam spacing over the run. In Fig.11 we show the resulting beampositions for a section of the run; the beams have been sorted so that the position is monotonically increasing as a function of beamnumber.

From Fig.11 it can be seen that there are areas of close beam spacing and then larger gaps; it is not clear how this will affect the profile reconstruction. The noisy beam data that is used is shown in Fig.12. Finally the reconstructed profile is shown in Fig.13 (solid line) along with the input profile (dashed). The reconstructed profile is very good - the input profile has not been exactly duplicated but the agreement is very good.

If we use a spacing of 0.4 m between pings then there is no beam overlap (however, the beam redundancy will reduce noise) and the resulting profile after the inversion process is the same as the backpropagated solution and has a resolution of 0.2 m.

II. Two-dimensional example

We now consider a two-dimensional reflectivity profile. We restrict ourselves to a small patch of the seabed in order to reduce the number of variables.The sidescan data is generated synthetically in the following manner. A beam is considered to be 20 cm wide. It is subdivided along its length into across-track patches; they are taken to correspond to 4 cm in length (in reality, it would be a time interval for which the length on the seabed would vary as cross-track range, but since we are considering a small patch of the seabed, we simply take the cross-track resolution to correspond to a fixed length). Each of the these 20 $\times$ 4 cm cells is then subdivided into 100 smaller patches and the reflectivity determined in each of these patches (using the known analytic form of the input reflectivity)and the 100 values of the reflectivity are summed to yield the integrated value for that cell. Also the centre coordinates for the 100 small patches are used to determine the entry in the matrix Ω relating the measured value to the pixels of the geocoded image. We will consider square pixels of 5 $\times$ 5 cm for the geocoded image. As for the one-dimensional case, the sonar is taken to have 5 beams; for the examples below, the sonar will advance 0.5 m between pings. The heading

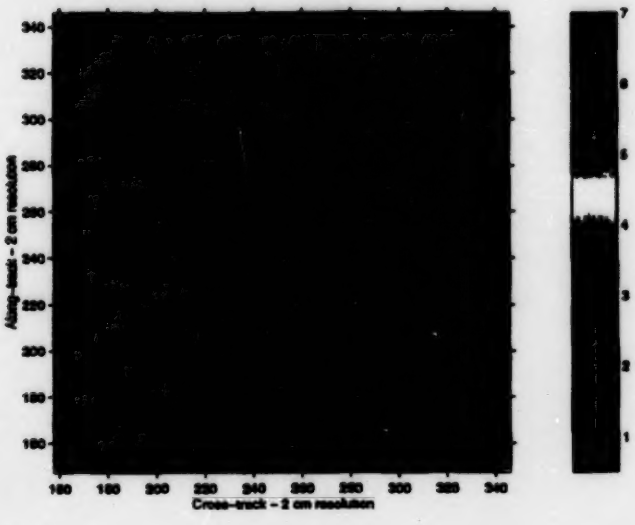

Figure 14: Input two-dimensional seabed reflectivity

of the towfish will also be allowed to vary in the simulations. This means the beams may overlap in some complicated pattern, particularly at the longer cross-track ranges; for example, a heading change of 0.1 degrees between pings would result in the beams overlapping approximately 0.1 m at 60 m range.

Our basic reflectivity patch consists of a background level of 0.5, a circle of 2 m diameter with value 2.0, a smaller circle of diameter 0.4 with value 4.0, and finally an even smaller circle of diameter 0.2 with value 3.0 superimposed on the other circles (i.e. giving a value of 7 in some places and 5 in a very small area) This input reflectivity is shown below in Fig.14. In the numerical simulation this patch is displaced 50 m in the Eastings direction.

Our approach to the inversion is basically the same as in the one-dimensional case. We consider 2135 measurements (7 pings × 5 beams × 61 time measurements) corresponding to the different beams and discretized time measurements which sample the object of Fig. 14. These are related to the desired pixels of the geocoded image (because we choose a pixel size of 5 cm × 5 cm there are in fact more pixels than measurements in this case) by the matrix Ω. The pixels corresponding to the two-dimensional image are rearranged as a large one-dimensional vector of unknowns. The "backpropagation" solution is given as before by

$$\vec{S}_{ap} = \frac{\Omega^T \vec{b}}{\delta \times \Omega^T \vec{1}} \tag{10}$$

Here we add a very small constant value to Ω everywhere in order that there are no divisions by zero. The non-linear estimation scheme is as before. The cost functional consists of the difference

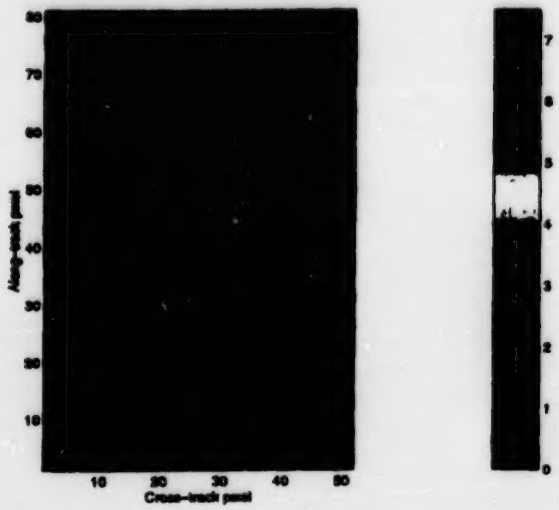

Figure 15: Reconstructed two-dimensional seabed reflectivity using backpropagation

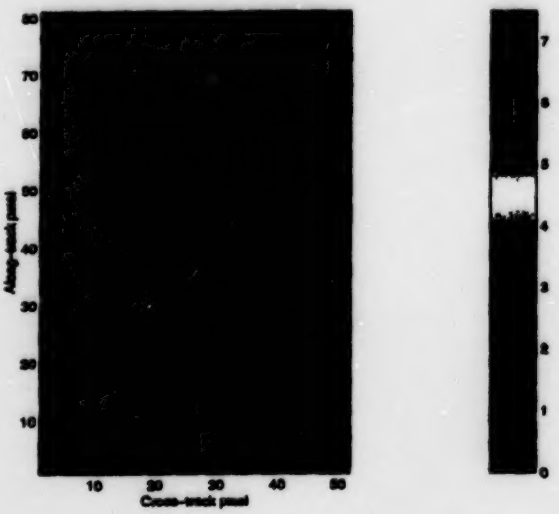

Figure 16: Reconstructed two-dimensional seabed reflectivity after non-linear iterations

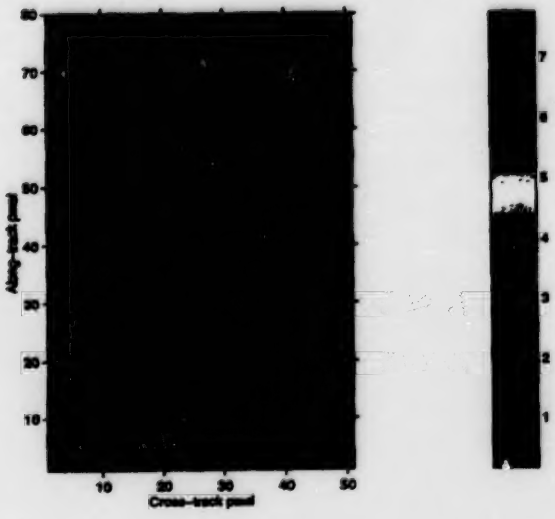

Figure 17: Reconstructed two-dimensional seabed reflectivity for heading variation

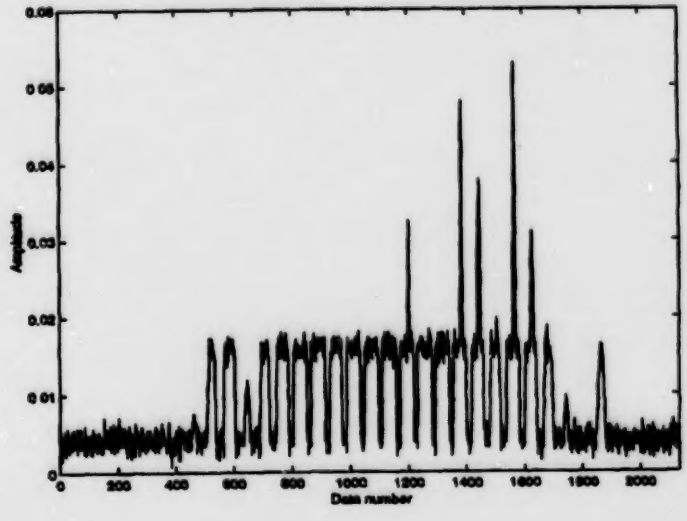

Figure 18: Noisy two-dimensional data (shown as one-dimensional vector)

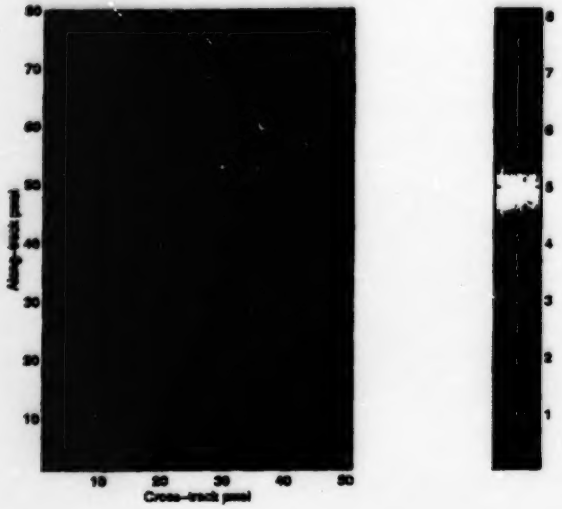

Figure 19: Reconstructed two-dimensional seabed reflectivity for heading variation with noisy data

between the predicted and real data and also a term constraining the variation of the solution; here for each pixel in the estimation we consider the square roots of the absolute values of its differences from the 4 connecting pixels in the two-dimensional image. In Fig. 15 we show the backpropagation result for no heading variation and in Fig. 16 the corresponding result when the backpropagated solution is iterated using the non-linear algorithm (51 iterations).

Next, we consider the same scenario but with the heading of the towfish changing by 0.03° at each ping; this means that the beams are additionally overlapping at sufficient cross-track range. In Fig. 17 we show the non-linear inversion using noiseless data and in Fig. 18 the corresponding result when 10% noise was added to the beamdata. The results are again very good. There are small differences between these results and those of Fig. 16 but more work is required to understand the effect of heading variation on the possible geocoded resolution.

SUMMARY

We have shown that the process of translating beam data into seabed reflectivity can be posed as an inverse problem. By utilizing overlapping beams, it is possible to improve the resolution of the system. The straightforward algebraic inversion for this problem can lead to unstable results for noisy data. Eliminating the small singular values improves the results, but perhaps a more promising approach is to use an iterative, non-linear optimization with a constraint on the oscillatory behaviour

of the solution. The algorithm was demonstrated with both one- and two-dimensional examples. The advantage of using simulated data is that one knows the "true" reflectivity profile. However, we hope to demonstrate in the future the advantages of using the reconstruction methods of this paper with real sidescan sonar data.

References

[1] L. Huff, "Evaluating beam utilization in the Klein system 5000 side scan sonar" *in Proceedings of Shallow Survey'99: International Conference on High Resolution Surveys in Shallow Water, October 18-20, 1999, Sydney, Australia*, editted by Roger Neill, DSTO Australia (1999).

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In this paper the conversion of sidescan sonar beam data into seabed reflectivity is posed as an inverse problem. The seabed reflectivity is discretized in terms of a basis set with unknown coefficients and values of these coefficients which yield the observed data, possibly with constraints on the behaviour of the reflectivity function, are sought. In particular, when the sonar's beamset from consecutive pings overlap each other, there is the possibility of improving the along-track resolution of the geocoded image over that of the nominal system beamwidth.

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